THE COMMON CORE STATE STANDARDS FOR MATHEMATICS

The U.S.A's Quest for Greater Focus and Coherence

SEAMEO RECSAM-University of Tsukuba Joint Seminar

Gabriel Matney
Associate Professor of Mathematics Education

BGSU | School of Teaching and Learning
BOWLING GREEN STATE UNIVERSITY

2ND GRADE MATHEMATICS PROBLEM

What are some ways you think 2nd graders would solve this problem?

There were 48 kindergarteners and first graders all together in the cafeteria. The kindergarteners left and 26 second graders came in. Now there are 54 students in the cafeteria. How many kindergarteners were in the cafeteria? How many first graders are in the cafeteria? [5]

The creation of this problem was support by the CORES Elementary grant funded by the Ohio Board of Regents (Matney, Bostic, & Brahier, 2012).

AGENDA

Brief Viewing of Early US Mathematics Education History

Brief Discussion of Learning from Other Nations

Basics of the Common Core State Standards

Curricular **Research** and Connections

INFLUENCE OF IDEAS

Archimedes, Socrates, Plato, Euclid

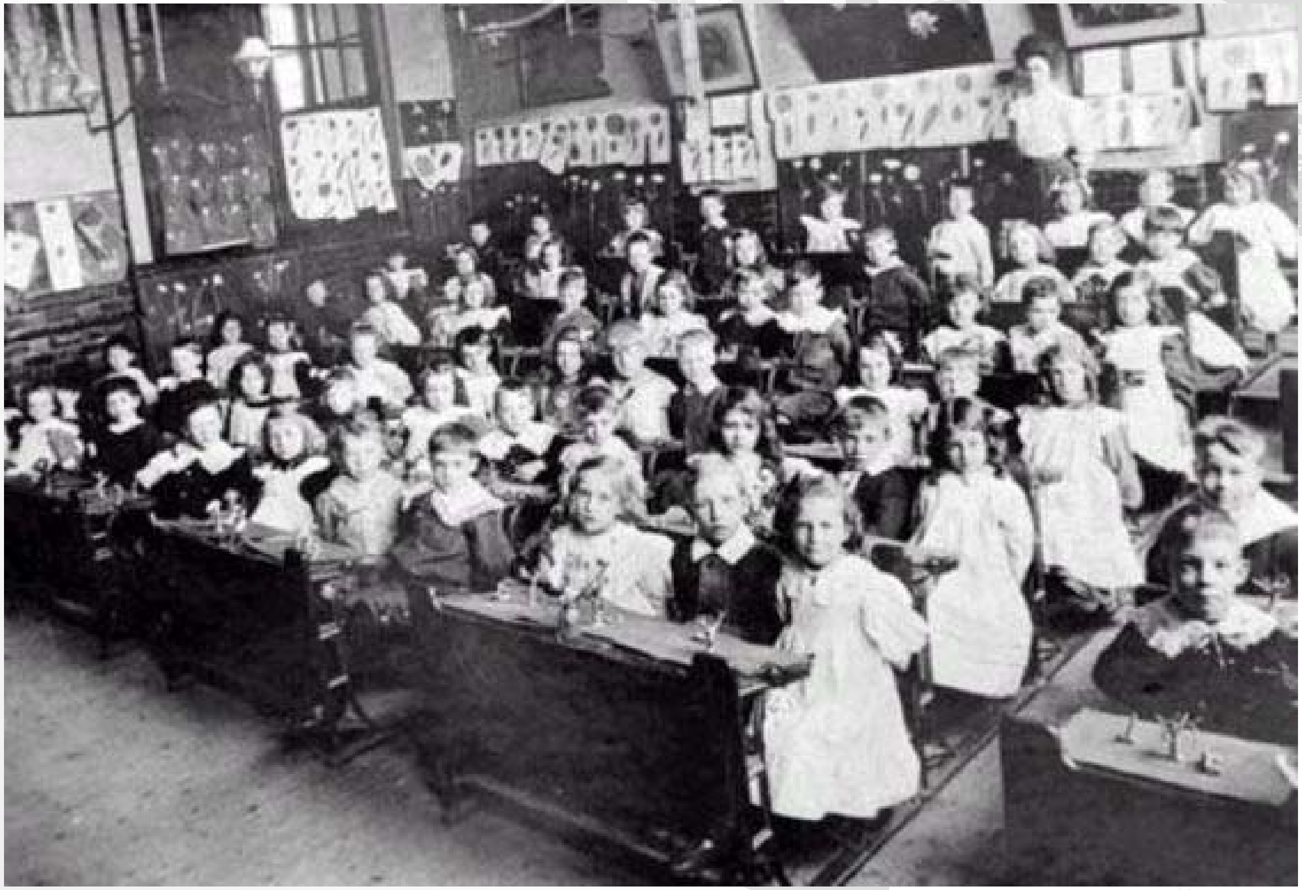
Rousseau
Emile pages 142 and 150, 1761

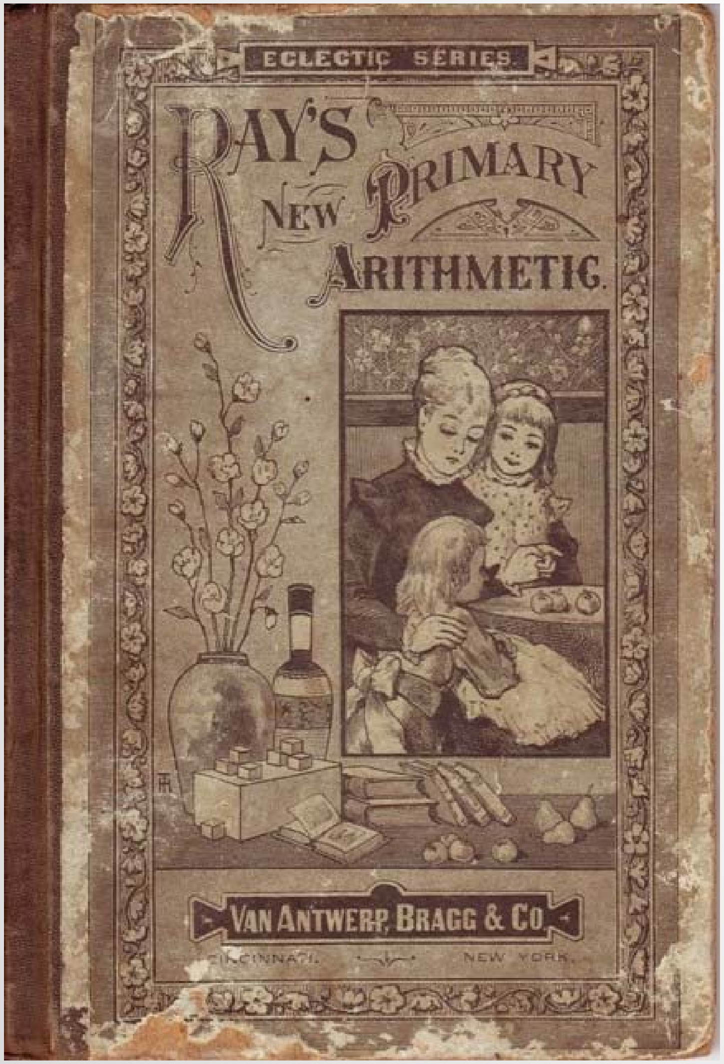
“Teach your scholar to observe the phenomena of nature; you will soon rouse his curiosity, but if you would have it grow, do not be in too great a hurry to satisfy this curiosity. Put the problems before him and let him solve them himself. Let him not be taught science, let him discover it. If ever you substitute authority for reason he will cease to reason; he will be a mere plaything of other people’s thoughts”

In 1842 the state of Connecticut passed a law that no child under the age of 15 could be employed without proof of first having attended school that year.

By 1918 all state's passed compulsory education laws requiring education for children.

More than 120-190 years ago mathematics education for reasoning was flourishing in the United States.

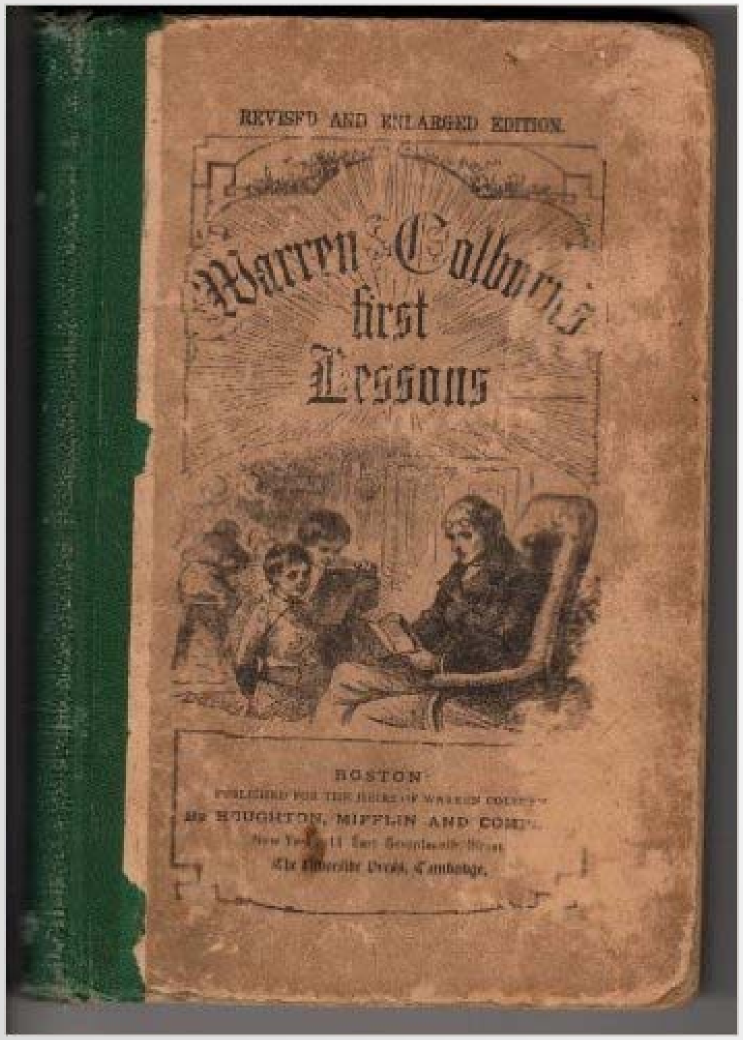




By 1834, Joseph Ray of Ohio had already published two books with his method of teaching through "analysis and induction".

Ray and other early American Mathematics Education Scholars like him believed that mathematics had value as a mental discipline.

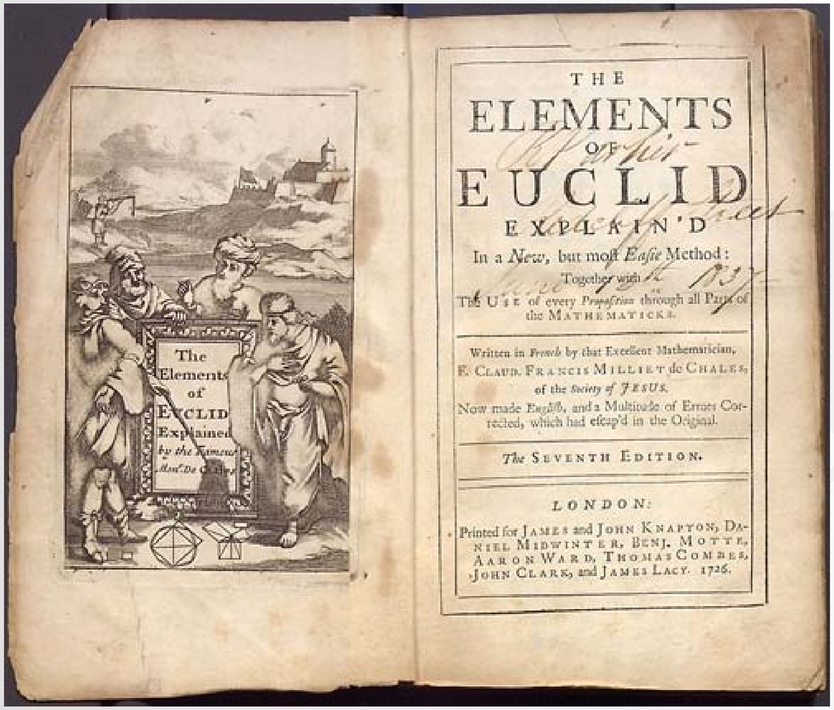
Ray said, "[To] be able to reason correctly, and to exercise, in all relations in life, the energies of a cultivated and disciplined mind is of more value that the mere attainment of a branch of knowledge…the pupil should not be taught merely to perform a certain routine of exercises mechanically, but to understand the ***why*** and the ***wherefore***" [7]



The ideas about mathematics education that were developing in the United States were being disseminated to other parts of the world.

Warren Colburn's "First Lessons in Arithmetic" spread across England and was translated into many languages in Europe.[1]

Calvin Wilson Mateer's book "Pen-calculation Arithmetic" was being distributed throughout China.[1]



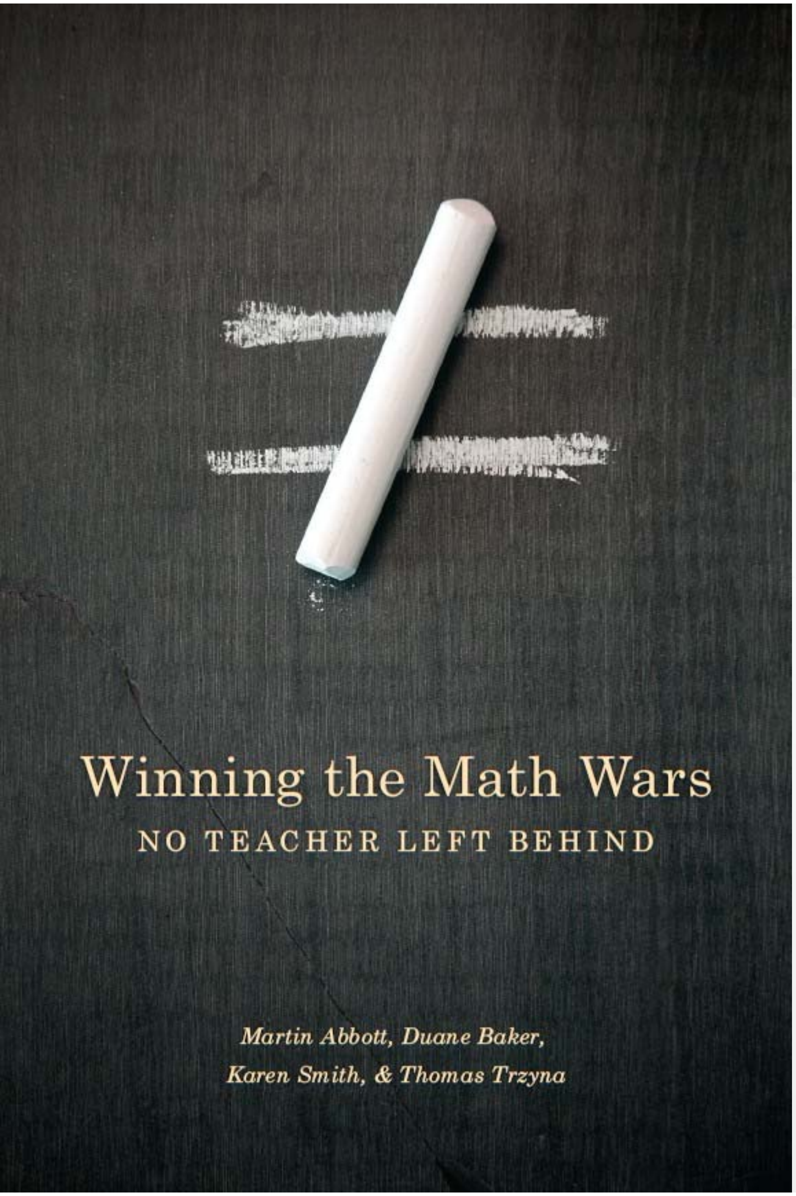
During this time American mathematics educators tended to combine arithmetic learning with a reasoning system.

Although difficult for students, these approaches encouraged both practical and theoretical thinking about mathematics.

Explored mathematics through the dual lenses of the questions:

How do we accomplish this calculation?

Why is it mathematically justified to do the calculation this way?



After this time of flourishing a debate began in the U.S.A about what kind of mathematical knowledge school children need.

- Some favored the "How" and others favored the "Why".

For complex reasons which include this debate about how and why, the overall advance of mathematics classroom instruction in the U.S.A stagnated in several ways, while other countries, such as Japan, Germany, and China, worked to improve teacher's instruction of mathematics.

There have always been strong pockets of research and innovation in mathematics and education, but systemically, teachers pedagogical practice in the classroom had not seen a substantial changed for many decades.

Learning from the success of other countries, such as Japan and Germany, it was possible for mathematics educators to have a renewed conversation with parents, teachers, policy makers, and politicians.

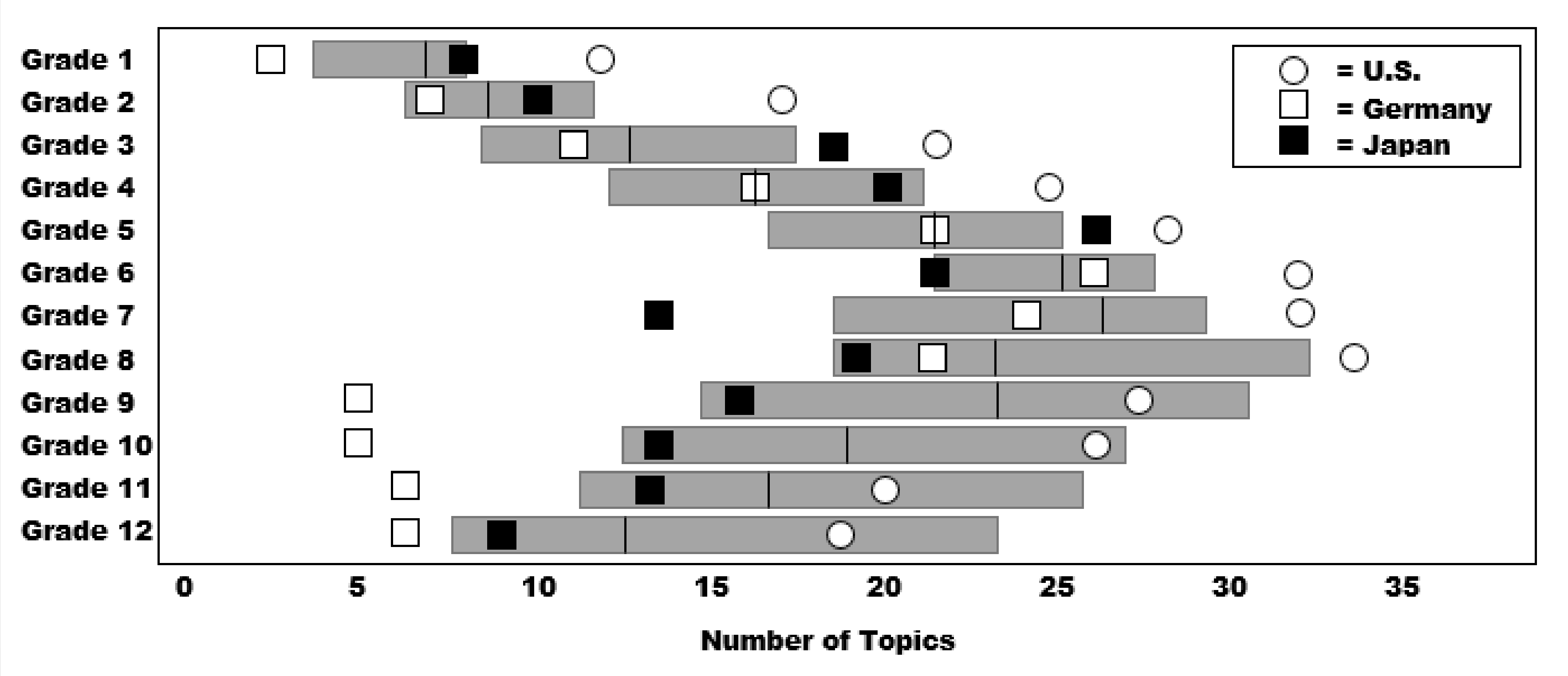


*Image recreated from Schmidt, McKnight, & Raizen (1997)[2]

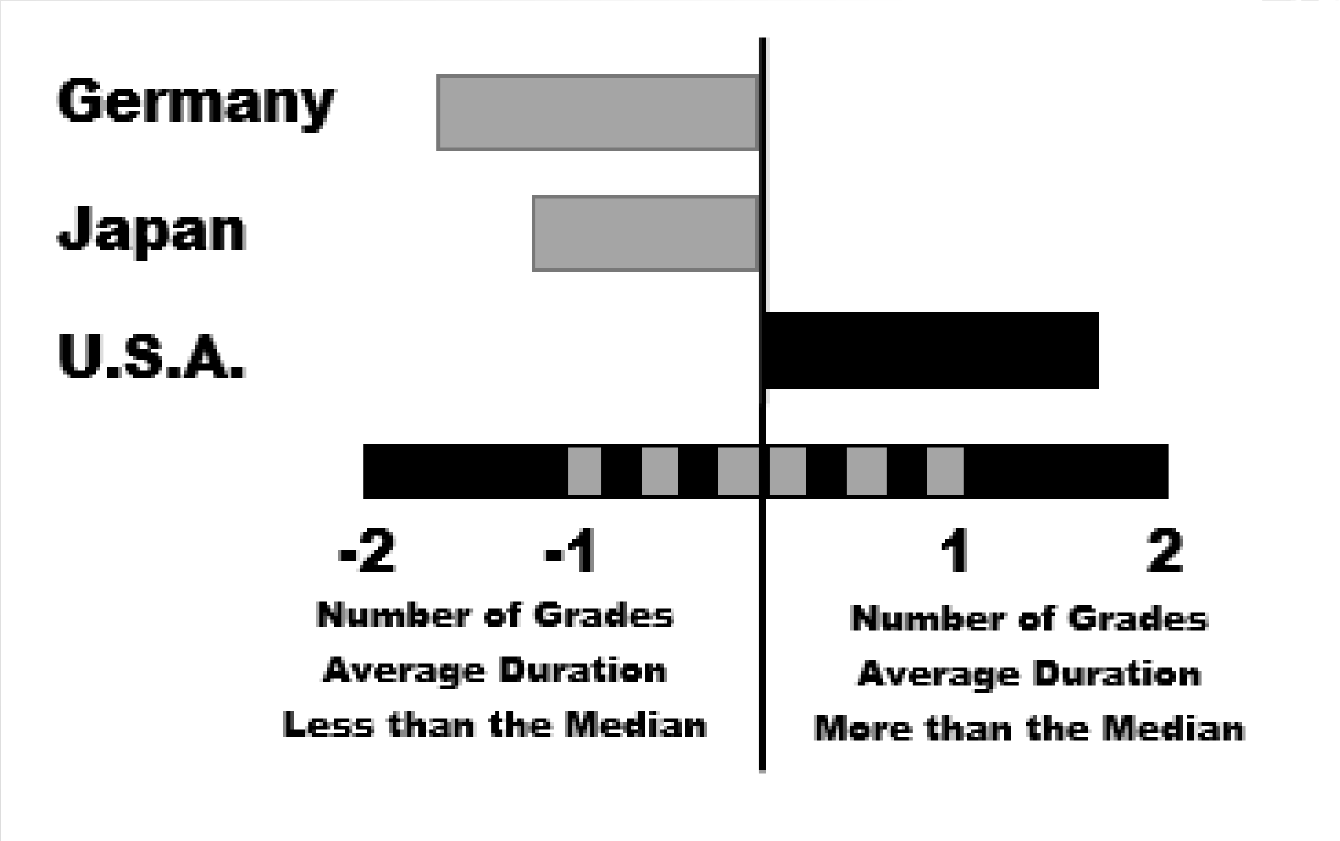


*Image recreated from Schmidt, McKnight, & Raizen (1997)[2]

Another stark difference between the U.S.A. and more successful nations dealt with the amount of time a topic spanned.

Instead of taking a topic and going very deep with it, U.S.A curriculum's typically spread it out over many grade levels.

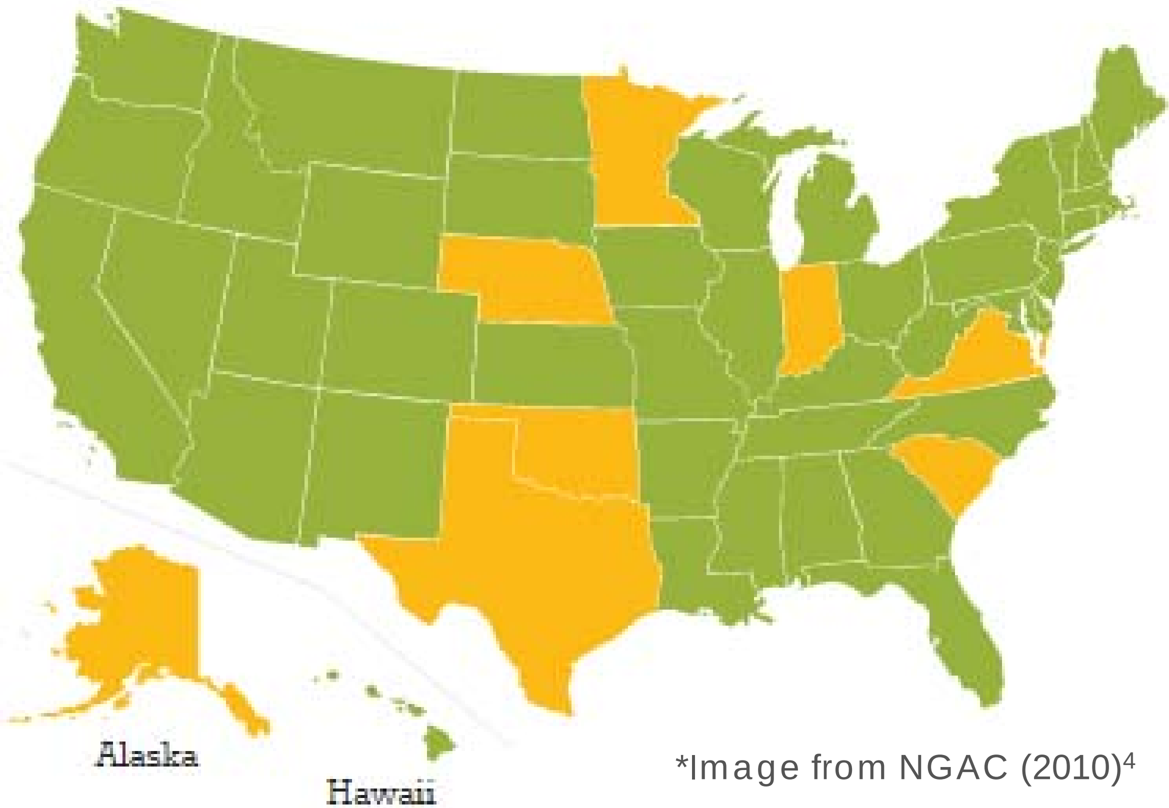


*Image from NGAC (2010)[4]

We also learned that our systems (educative, governmental, and political) contributed to the fragmenting of the curricula through the fact that textbooks were made to be overly inclusive of many topics so they could "cover" the expectations of many states.[2]

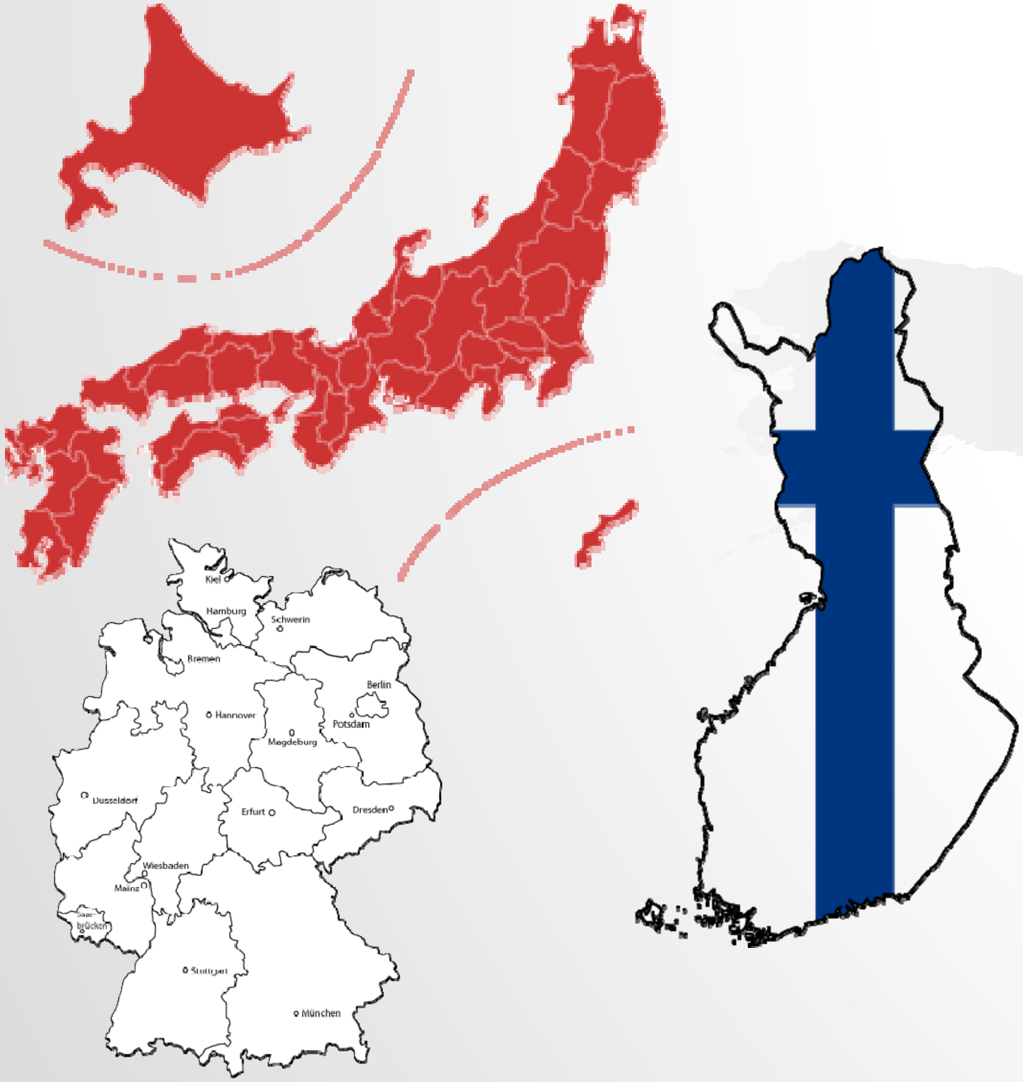
Though many reformers had been discussing these notions as problematic prior to these comparison, by the late 1990's the U.S.A. was more broadly informed that we suffered from unfocused curricula along with incoherent and overly inclusive textbooks.

The U.S.A began a quest to find focus and coherence in the mathematics that was taught during grades K-12, across the various states and territories.

National Governors Association

CCSSO
Council of Chief State School Officers

In 2009 the NGA and CCSSO organized and working groups began the design the Common Core State Standards for Mathematics (CCSSM) based on research about the world's best education systems, input from teachers, and preexisting US state standards. [4]



Research about other nations and their results on international exams would served as a basis for standards reform:

For example, **TIMSS**, **PISA**, and the ***The Teaching Gap***[6] ensured that ideas from around the world would influence the making of the CCSSM.

After the CCSSM were designed "expert mathematics content analysts conducted a side-by-side comparison" of the CCSSM and the Japanese Mathematics Course of Study.[3]

Three Areas of Comparison

Rigor – "refers to the degree that sets of standards address key content that prepares students for success beyond high school"[3]

Coherence – "refers to whether the standards reflect a meaningful structure, revealing significant relationships among topics and suggest a logical progression of content and skills over the years."[3]

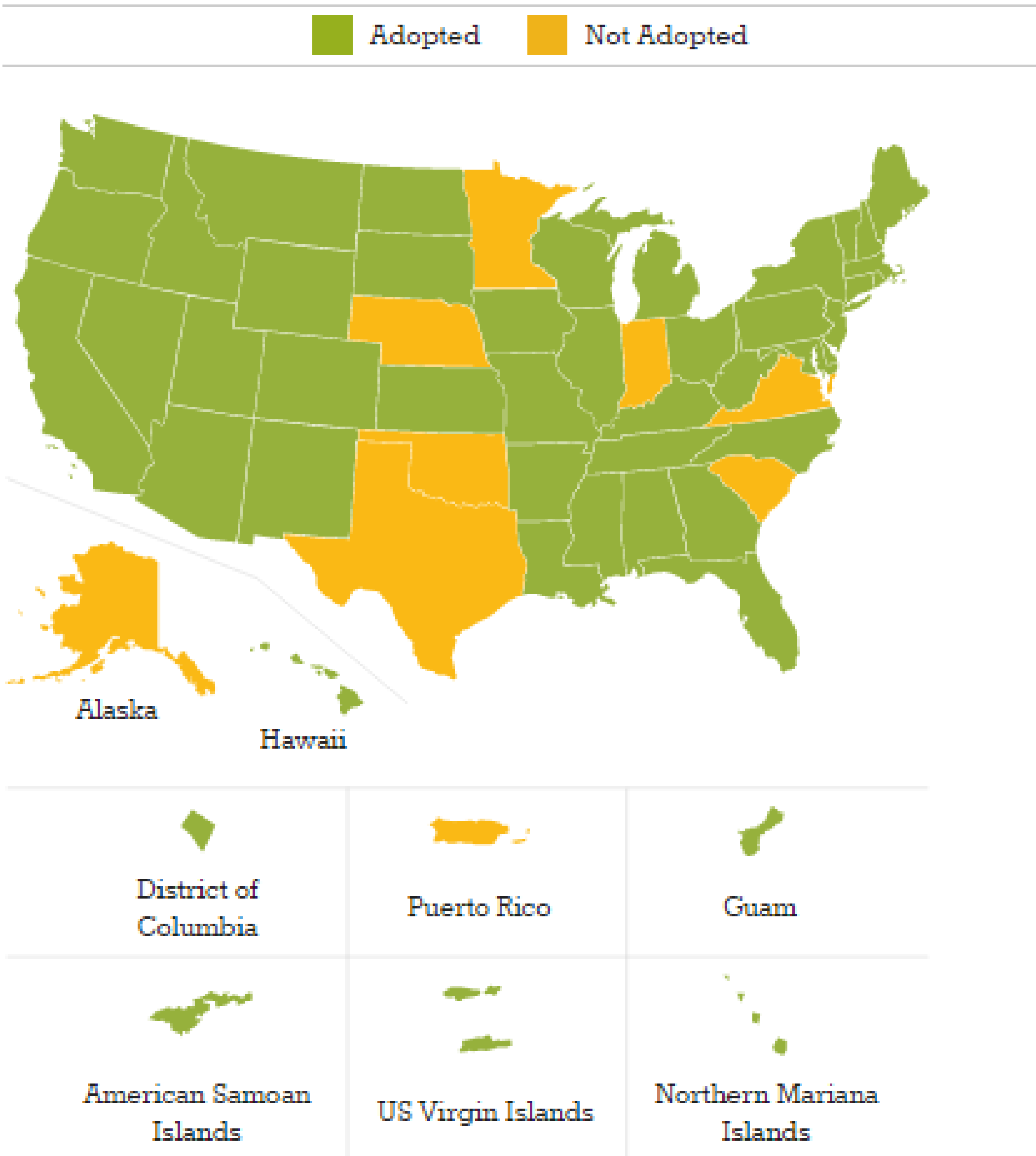
Focus – "refers to whether the standards suggest an appropriate balance in conceptual understanding, procedural skill, and problem solving with an emphasis on application and modeling."[3]

Major Findings

Comparable Rigor – Some content given at different grades but usually within a year.[3]

Comparable Coherence – Share key traits. Both describe coherent expectations through grade 12.[3]

Comparable Focus – Both describe focused expectations. At some points the CCSSM gives more detail and specificity about content balance.[3]



ADOPTION OF THE COMMON CORE STATE STANDARDS FOR MATHEMATICS

*Image from NGAC (2010)[4]

How to read the grade level standards

Standards define what students should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

Domain

Number and Operations in Base Ten 3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

Standard

Cluster

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

*Image from NGAC (2010)[4]

STANDARDS

Define what students should understand and be able to do and if possible specify the grade level at which it should be done.

In the USA, standards are developed and decided upon at state level. The CCSSM represents many states coming together to make the same set of standards.

CURRICULUM

May be thought of as a number of different ideas, but in this case, it is associated with the content organization and the pedagogical pathways planned and enacted by districts and teachers. These actions should be based on the content and processes given in the standards.

In the USA, curriculum decisions are made at the district, school, and teacher level. Though the CCSSM organizes content by grades, it doesn't dictate order beyond that and it doesn't state how teachers are to implement.

THE CCSSM INFORMS CURRICULUM BUT DOES NOT DEFINE IT

CCSSM

Standards for Mathematics **Content**

Standards for Mathematical **Practice**

National Research Council

Adaptive Reasoning
Strategic Competence
Conceptual Understanding
Procedural Fluency
Productive Disposition

National Council of Teachers of Mathematics

Communication
Connection
Problem Solving
Reasoning and Proof
Representation

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. **Reason abstractly and quantitatively.**
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. **Construct viable arguments and critique the reasoning of others.**
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. **Model with mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. **Use appropriate tools strategically.**
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. **Attend to precision.**
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical **Practice**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. **Look for and make use of structure.**
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. **Look for and express regularity in repeated reasoning.**

Fluency and the CCSSM

In order to take a closer look at the curricular influence of the CCSSM on one topic area that has been a struggle for children in the U.S.A. I would like to share some of my research with teachers and children from the U.S.A. after the CCSSM.

A Major Difficulty: **Procedural Fluency**

Procedural fluency is the skill in carrying out procedures flexibly, accurately, efficiently and appropriately.[4]

Fluency and the CCSSM

NCTM Principals and Standards emphasize *fluency* as an important aspect of mathematics learning, and its included in both the Standards for Mathematics Content and the Standards for Mathematical Practice.

Students who are mathematically fluent in the elementary grades give attention to accuracy, have flexibility in thinking, and develop efficiencies for working with numbers and arithmetic operations.[5]

How does the structure of CCSSM help with fluency?

Fluency and the CCSSM

The CCSSM mentions fluency 25 times

Fluency should not be confused with rote recall of facts from memorization, or dictating timed tests.

CCSSM lead author writes, “It is no accident that the standard says ‘know from memory’ rather than “memorize”. The first describes an outcome, whereas the second might be seen as describing a method of achieving that outcome. So no, the standards are not dictating timed tests.” (October 31, 2011)

Grade	Standard	Addition	Subtraction	Multiplication	Division
K	Fluency K.OA.5	Numbers within 1 to 5			
K	Solving K.OA.2 & K.OA.3	Solve word problems by using objects or drawings and decompose numbers less than or equal to 10			
K	Modeling Ideas K.NBT.1	Use objects or drawing to compose and decompose numbers from 11 to 19 into tens and ones			
1	Fluency 1.OA.6 & 1.NBT.5	Numbers within 10 Mentally find 10 more or 10 less than a two-digit number			
1	Solving 1.OA.1	Solve word problems using various strategies for numbers within 20, including problems with 3 whole numbers, by using objects, drawings, and equations	Solve word problems using various strategies for numbers within 20, by using objects, drawings, and equations		
1	Modeling Ideas 1.OA.2	Use models, properties, and place value strategies to add a two-digit number to a one-digit number or add a two-digit number to a multiple of 10	Use models, properties, and strategies to subtract one multiple of 10 from another multiple of ten in the range of 10 to 90.		
2	Fluency 2.NBT.5 & 2.OA.2	Use strategies of place value and properties to add and subtract numbers within 100 Mentally add and subtract numbers within 20, know from memory all sums of two one-digit numbers			
2	Solving 2.OA.1	Solve word problems involving one or two steps for numbers within 100 by using drawings and equations			
2	Modeling Ideas 2.NBT.7 & 2.NBT.8 & 2.OA.4	Use models, properties, and place value to add and subtract numbers within 1000 Mentally add or subtract 10 or 100 to a number between 100 and 900		Use addition to find the number of objects arranged in rectangular arrays, 5x5 or smaller.	

Fluency and the CCSSM

[5]*Matney (2014). Teaching Children Mathematics, "Early Mathematics Fluency with the CCSSM"*

Fluency and the CCSSM

Grade		Addition	Subtraction	Multiplication	Division
3	Fluency 3.NBT.2 & 3.OA.7	Numbers within 1,000 using strategies and algorithms based on place value, properties, and the relationship between addition and subtraction		Numbers within 100	
				Know all products of two one-digit numbers	
	Solving 3.OA.3 & 3.OA.8	Solve two-step word problems using addition, subtraction, multiplication and division			
				Solve problems using various strategies for numbers within 100 by using drawings and equations	
	Modeling Ideas 3.OA.9 & 3.OA.1 & 3.OA.2	Identify arithmetic patterns and explain using properties of operations			
				Interpretations of products and quotients	
4	Fluency 4.NBT.4	Fluently add and subtract using the standard algorithm for numbers less than or equal to 1,000,000			
	Solving 4.OA.2 & 4.OA.3	Solve multi-step word problems using addition, subtraction, multiplication and division, including problems in which remainders must be interpreted			
				Solving problems involving multiplicative comparison	
	Modeling Ideas 4.OA.5 & 4.NBT.5 & 4.NBT.6	Generate and analyze patterns			
				Use strategies of place value and properties to multiply a four digit number by a one-digit number, and multiply two two-digit numbers. Explain using equations, arrays, and/or area models	Use strategies of place value and properties to divide up to a four-digit number by a one-digit number. Explain using equations, arrays, and/or area models
5	Fluency 5.NBT.5			Multiply multi-digit whole numbers using the standard algorithm	
	Modeling Ideas 5.NBT.6				Use strategies and properties to divide whole numbers with up to four digits by two-digit numbers. Explain using equations, arrays, and/or area models

[5] *Matney (2014). Teaching Children Mathematics, "Early Mathematics Fluency with the CCSSM"*

Fluency and the CCSSM

Fluency through Focused Instruction
Number of the Day

Today's number is 29. Write any number sentence you want to start. Then come up with another number sentence using those same numbers. [5]

Fluency and the CCSSM

Fluency through Focused Instruction
Number of the Day

We have Marcia's idea of 22 + 7 = 29. Can anyone besides Marcia share one other number sentence she might have written?

[*Denzell*] She could write it 27 + 2 = 29. It's easier to add 2 on 27 than 7 on 22.

[*Marcia*] I don't think that will work. That would take part of the number and move it instead of the whole thing. Like in 7 + 22 = 29?

[*Mary*] Hmm, I see Denzell's idea works but like we were talking before, I don't know if it's a strategy. [5]

Fluency and the CCSSM

Fluency through Focused Instruction
Number of the Day

So Mary is bringing up another thing we have been considering. Does an idea work in all situations? In your partner pairs consider Denzell's idea and try to give reasons for why it will or will not always work. [*After a few minutes work*] **Let's come together and share ideas.**

[*John*] We found that Denzell is trading the two numbers and that works for others. [*Writes 23 + 6 = 29 and 26 + 3 = 29 on the white board*]

[*Mary*] We also think it always works because he is not changing how much it is all together. It's just the ones. [5]

Fluency and the CCSSM

Fluency through Focused Instruction
Number of the Day

[*Kendrae*] We switched the numbers but we expanded first. Can we show? [*Teacher motions for Kendrae to write it on the board*] So we wrote Denzell's way. [*He writes* $20 + 2 + 7 = 20 + 7 + 2$] 22 is really 20 + 2. Then add 7 more. Or we can switch when we add the 2 and 7. It will always work because he is adding in a different order but adding the same numbers.

[*Marcia*] Oh ya I can see that now. [5]

Fluency and the CCSSM

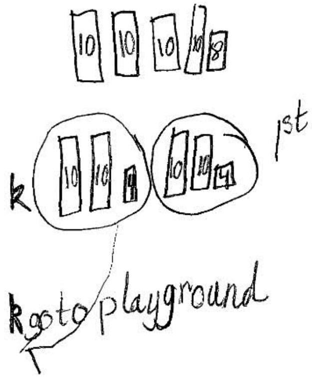
Fluency through Problem Solving

There were 48 kindergarteners and first graders all together in the cafeteria. The kindergarteners left and 26 second graders came in. Now there are 54 students in the cafeteria. How many kindergarteners were in the cafeteria? How many first graders are in the cafeteria? [5]

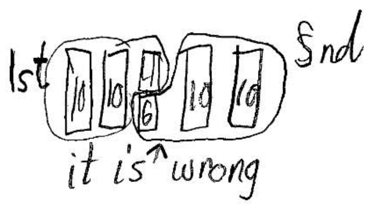
The creation of this problem was support by the CORES Elementary grant funded by the Ohio Board of Regents (Matney, Bostic, & Brahier, 2012).

Fluency and the CCSSM

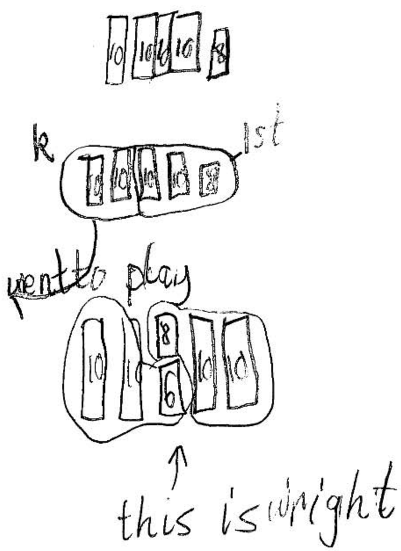
[5]Fluency through Problem Solving



"We first tried it out with blocks and then drew pictures. Since there were 48 we made 48 like this 4 tens and 8. Then took 24 kindergarteners and 24 first graders. But the kindergarteners left, yeah, we are going to the play ground!"



"Then there were 24 first graders and 26 second graders. But that was only 50, not 54, there are 5 tens. It was wrong. We tried many more times, like with 26 and 22 kindergarteners, and we saw we needed to take out less kindergarteners to get to 54."



"We found that 20 kindergarteners went to the playground, yeah were going to the playground! Then when the second grade came in they added 26 to the 28. 6 and 8 make 14 and 40 plus 14 is 54. That's how we do it."

Fluency and the CCSSM

[5]Fluency through Problem Solving

Step 1	Step 2	Step 3
54 first & secened	48 Kindergarden & first	20 kindergarden
−26 secend	−28 first	+28 first
28 first	20 Kindergarden	48 kindergarden & first

"We did it by going backwards. There were 54 at the end and 26 were second graders. So taking 26 away, we said, 54, 44, 34, and six more was 28 first graders in step one. Then at the beginning there were 48 and 28 were first graders. So 48, 38, 28, and eight more was 20 kindergateners in step two. So that's how we got the answer in step three."

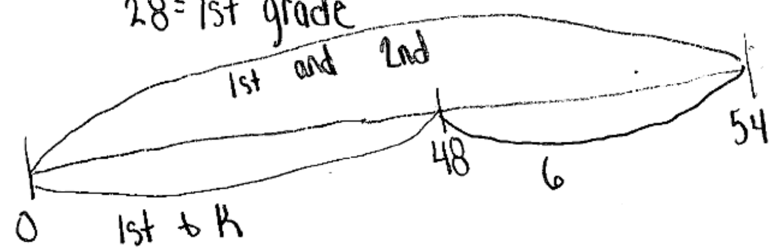
Fluency and the CCSSM

[5]Fluency through Problem Solving

Step 1	Step 2		Step 3
$\begin{array}{r} 54 \\ -\ 48 \\ \hline 6 \end{array}$	$\begin{array}{r} 26 \\ -\ 20 \\ \hline 6 \end{array}$	$\begin{array}{r} 48 \\ -\ 20 \\ \hline 28 \end{array}$	$\begin{array}{r} 28 \\ +\ 26 \\ \hline 54 \end{array}$

6 = how much more after 2nd grade came.

20 = Kindergarten
26 = Second grade
28 = 1st grade



"Since the kindergarteners left and the second grade came in (pause) the total went up by 6 so there are 6 more second graders. So there must be 20 kindergarteners, see, 26 minus 20 is 6. If there were 48 at the start, 20 were kindergarten, then 48 minus 20 is 28 first graders. I don't know why we did step three I don't think we need it."

Fluency and the CCSSM

Fluency through Modeling and Representing New Ideas

[5]Mrs. Owens had been engaging her students in working through double digit addition problems using base 10 blocks as a model. When they were ready she posed this question to them. “Is there a way you can add the amounts without the blocks?” [5]

Fluency and the CCSSM

[5]Fluency through Modeling and Representing New Ideas

```
  27
+ 35
----
  12
  50
----
  62
```

"7 plus 5 is 12 so I wrote it down there. But this 2 is really 20, right? and 3 is really 30, so 20 plus 30 is 50. Then I wrote it down too. The one (points to 12) means 10 and 5 (points to the 50) means 50, so 10 and 50 is 60 plus the 2 more."

Fluency and the CCSSM

[5]Fluency through Modeling and Representing New Ideas

"I did this the same but I messed up. (Mrs. Owens: ok, explain what you did) I added 6 and 8, 14, then 40 and 50, 90. At first I saw 90 and 10 and wrote 91, but I knew that wasn't right, so I scratched it out and wrote 100, then added 4 more, 104."

Fluency and the CCSSM – Focus and Coherence

1) Give students tasks like Number Talks, Problem of the Day, and modeling new thinking to develop number relationships, accuracy, flexibility, and efficiency.[5]

2) Use Problem Solving within the *fluency* range in order to practice and give more experiences with number and operations.[5]

3) Allow children the space to model mathematical ideas, and when its time, challenge them to use their modeling ideas with newer, more sophisticated problems.[5]

QUESTIONS?

gmatney@bgsu.edu

Gabriel Matney
Associate Professor of
Mathematics Education

BGSU | School of Teaching and Learning
BOWLING GREEN STATE UNIVERSITY

NOTES

[1]Ma, L. (2010). *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*. New York, NY: Routledge.

[2]Schmidt, W. H., McKnight, C. C., & Raizen, S. A. (1997). A Splintered Vision: An Investigation of U.S. Science and Mathematics Education. Norwell, MA: Kluwer.

[3]Achieve (2010). Comparing the Common Core State Standards and Japan's Mathematics Curriculum Course of Study. Retrieved on February 9th, 2016 at: www.achieve.org

[4]National Governors Association Center for Best Practices (NGAC), Council of Chief State School Officers (CCSSO). (2010). *Common Core State Standards for Mathematics.* Washington, DC: Author. http://www.corestandards.org.

[5]Matney, G. (2014). Early Mathematics Fluency with the CCSSM. *Teaching Children Mathematics 21*(1), 26-35. Reston, VA: National Council of Teachers of Mathematics.

[6]Stigler, J. W. and Hiebert, J. (1999) *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom. New York, NY:* Free Press.

Kullman, D. (1997). *Joseph Ray: The McGuffey of Mathematics.* Paper submitted to the Ohio Journal of Mathematics. Columbus, OH.